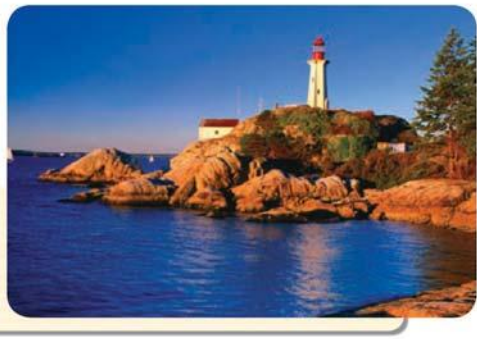
7.6 Apply the Sine and Cosine Ratios



Before You used the tangent ratio.

Now You will use the sine and cosine ratios.

Why So you can find distances, as in Ex. 39.

Key Vocabulary
- **sine**
- **cosine**
- **angle of elevation**
- **angle of depression**

The **sine** and **cosine** ratios are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

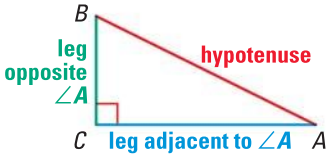
KEY CONCEPT *For Your Notebook*

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written $\sin A$ and $\cos A$) are defined as follows:

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

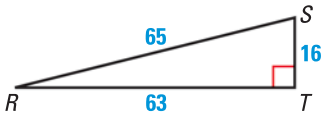
$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$



ABBREVIATE
Remember these abbreviations:
sine → sin
cosine → cos
hypotenuse → hyp

EXAMPLE 1 Find sine ratios

Find $\sin S$ and $\sin R$. Write each answer as a fraction and as a decimal rounded to four places.



Solution

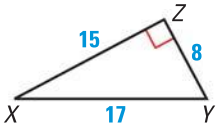
$$\sin S = \frac{\text{opp. } \angle S}{\text{hyp.}} = \frac{RT}{SR} = \frac{63}{65} \approx 0.9692$$

$$\sin R = \frac{\text{opp. } \angle R}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{65} \approx 0.2462$$

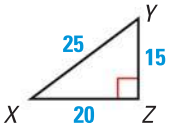
✓ GUIDED PRACTICE for Example 1

Find $\sin X$ and $\sin Y$. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

1.

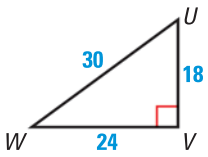


2.



EXAMPLE 2 Find cosine ratios

Find cos U and cos W. Write each answer as a fraction and as a decimal.



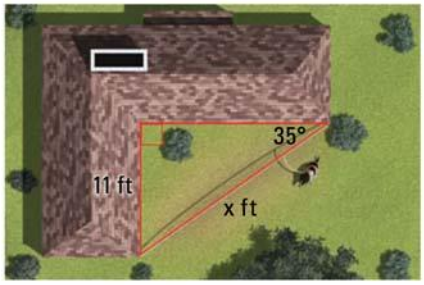
Solution

$$\cos U = \frac{\text{adj. to } \angle U}{\text{hyp.}} = \frac{UV}{UW} = \frac{18}{30} = \frac{3}{5} = 0.6000$$

$$\cos W = \frac{\text{adj. to } \angle W}{\text{hyp.}} = \frac{WV}{UW} = \frac{24}{30} = \frac{4}{5} = 0.8000$$

EXAMPLE 3 Use a trigonometric ratio to find a hypotenuse

DOG RUN You want to string cable to make a dog run from two corners of a building, as shown in the diagram. Write and solve a proportion using a trigonometric ratio to approximate the length of cable you will need.



Solution

$$\sin 35° = \frac{\text{opp.}}{\text{hyp.}}$$ Write ratio for sine of 35°.

$$\sin 35° = \frac{11}{x}$$ Substitute.

$$x \cdot \sin 35° = 11$$ Multiply each side by x.

$$x = \frac{11}{\sin 35°}$$ Divide each side by sin 35°.

$$x \approx \frac{11}{0.5736}$$ Use a calculator to find sin 35°.

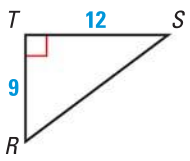
$$x \approx 19.2$$ Simplify.

▸ You will need a little more than 19 feet of cable.

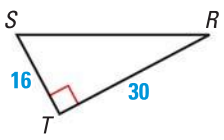
✓ GUIDED PRACTICE for Examples 2 and 3

In Exercises 3 and 4, find cos R and cos S. Write each answer as a decimal. Round to four decimal places, if necessary.

3.



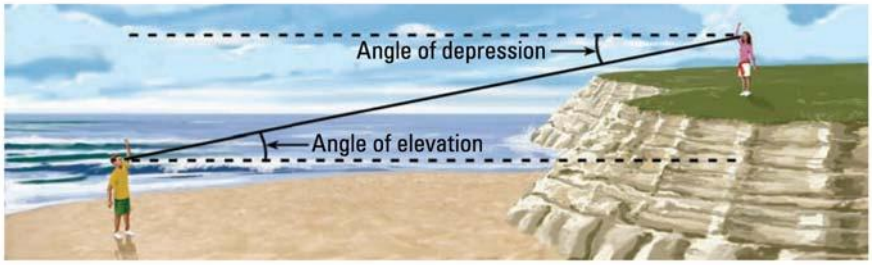
4.



5. In Example 3, use the cosine ratio to find the length of the other leg of the triangle formed.

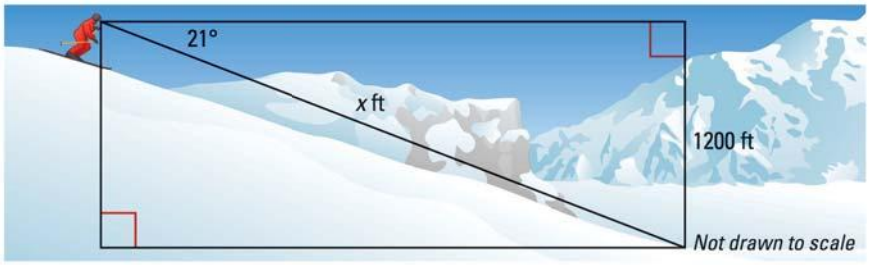
ANGLES If you look up at an object, the angle your line of sight makes with a horizontal line is called the **angle of elevation**. If you look down at an object, the angle your line of sight makes with a horizontal line is called the **angle of depression**.

APPLY THEOREMS
Notice that the angle of elevation and the angle of depression are congruent by the Alternate Interior Angles Theorem on page 155.



EXAMPLE 4 Find a hypotenuse using an angle of depression

SKIING You are skiing on a mountain with an altitude of 1200 meters. The angle of depression is 21°. About how far do you ski down the mountain?



Solution

$$\sin 21^\circ = \frac{\text{opp.}}{\text{hyp.}}$$ Write ratio for sine of 21°.

$$\sin 21^\circ = \frac{1200}{x}$$ Substitute.

$$x \cdot \sin 21^\circ = 1200$$ Multiply each side by x.

$$x = \frac{1200}{\sin 21^\circ}$$ Divide each side by sin 21°.

$$x \approx \frac{1200}{0.3584}$$ Use a calculator to find sin 21°.

$$x \approx 3348.2$$ Simplify.

▶ You ski about 3348 meters down the mountain.

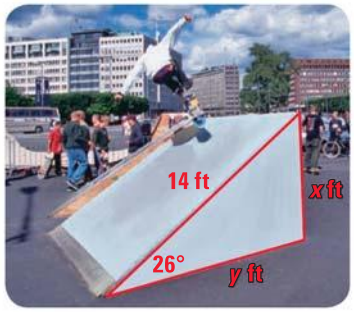
Animated Geometry at classzone.com

GUIDED PRACTICE for Example 4

6. **WHAT IF?** Suppose the angle of depression in Example 4 is 28°. About how far would you ski?

EXAMPLE 5 Find leg lengths using an angle of elevation

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and length of the base of the ramp.



ANOTHER WAY
For alternative methods for solving the problem in Example 5, turn to page 481 for the **Problem Solving Workshop**.

Solution

STEP 1 **Find** the height.

$$\sin 26° = \frac{\text{opp.}}{\text{hyp.}}$$ Write ratio for sine of 26°.

$$\sin 26° = \frac{x}{14}$$ Substitute.

$$14 \cdot \sin 26° = x$$ Multiply each side by 14.

$$6.1 \approx x$$ Use a calculator to simplify.

▶ The height is about 6.1 feet.

STEP 2 **Find** the length of the base.

$$\cos 26° = \frac{\text{adj.}}{\text{hyp.}}$$ Write ratio for cosine of 26°.

$$\cos 26° = \frac{y}{14}$$ Substitute.

$$14 \cdot \cos 26° = y$$ Multiply each side by 14.

$$12.6 \approx y$$ Use a calculator to simplify.

▶ The length of the base is about 12.6 feet.

EXAMPLE 6 Use a special right triangle to find a sine and cosine

Use a special right triangle to find the sine and cosine of a 60° angle.

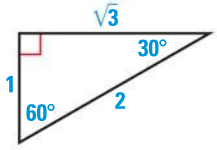
DRAW DIAGRAMS
As in Example 4 on page 468, to simplify calculations you can choose 1 as the length of the shorter leg.

Solution

Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and 2. Then set up sine and cosine ratios for the 60° angle.

$$\sin 60° = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$

$$\cos 60° = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{2} = 0.5000$$



GUIDED PRACTICE for Examples 5 and 6

7. **WHAT IF?** In Example 5, suppose the angle of elevation is 35°. What is the new height and base length of the ramp?

8. Use a special right triangle to find the sine and cosine of a 30° angle.

7.6 EXERCISES

HOMEWORK KEY

- ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 33
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 18, 29, 35, and 37
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

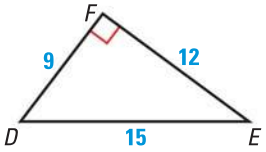
SKILL PRACTICE

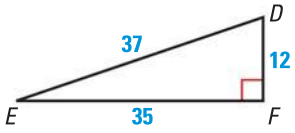
1. **VOCABULARY** Copy and complete: The sine ratio compares the length of _?_ to the length of _?_.

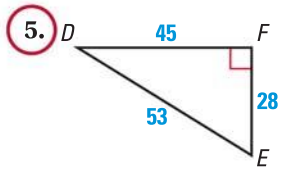
2. ★ **WRITING** *Explain* how to tell which side of a right triangle is adjacent to an angle and which side is the hypotenuse.

EXAMPLE 1 on p. 473 for Exs. 3–6

FINDING SINE RATIOS **Find sin *D* and sin *E*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.**

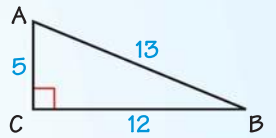
3. 

4. 

5. 

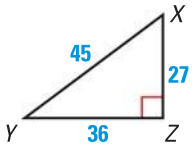
6. **ERROR ANALYSIS** *Explain* why the student's statement is incorrect. Write a correct statement for the sine of the angle.

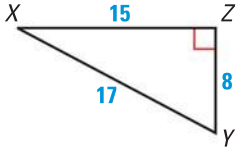
$\sin A = \frac{5}{13}$

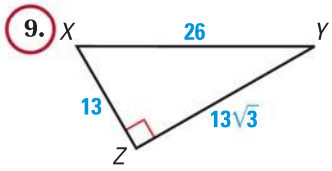


EXAMPLE 2 on p. 474 for Exs. 7–9

FINDING COSINE RATIOS **Find cos *X* and cos *Y*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.**

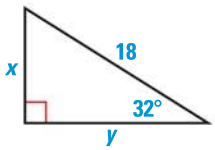
7. 

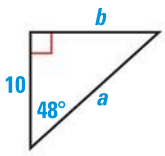
8. 

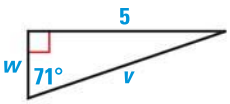
9. 

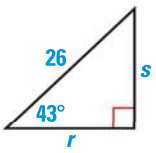
EXAMPLE 3 on p. 474 for Exs. 10–15

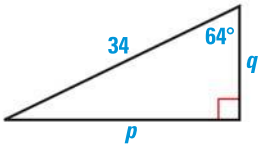
USING SINE AND COSINE RATIOS **Use a sine or cosine ratio to find the value of each variable. Round decimals to the nearest tenth.**

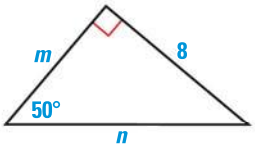
10. 

11. 

12. 

13. 

14. 

15. 

EXAMPLE 6 on p. 476 for Ex. 16

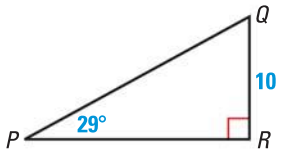
16. **SPECIAL RIGHT TRIANGLES** Use the 45°-45°-90° Triangle Theorem to find the sine and cosine of a 45° angle.

17. ★ **WRITING** *Describe* what you must know about a triangle in order to use the sine ratio and the cosine ratio.

18. ★ **MULTIPLE CHOICE** In $\triangle PQR$, which expression can be used to find PQ?

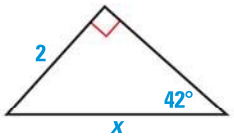
Ⓐ $10 \cdot \cos 29°$ Ⓑ $10 \cdot \sin 29°$

Ⓒ $\frac{10}{\sin 29°}$ Ⓓ $\frac{10}{\cos 29°}$

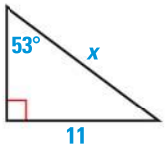


ALGEBRA **Find the value of x. Round decimals to the nearest tenth.**

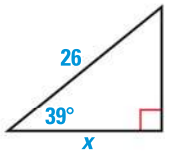
19.



20.

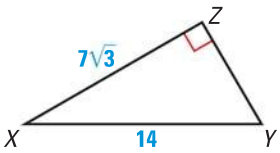


21.

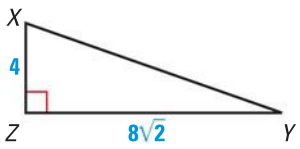


FINDING SINE AND COSINE RATIOS **Find the unknown side length. Then find $\sin X$ and $\cos X$. Write each answer as a fraction in simplest form and as a decimal. Round to four decimal places, if necessary.**

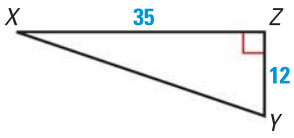
22.



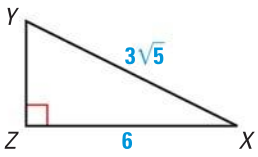
23.



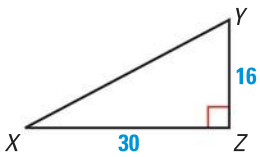
24.



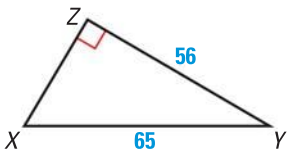
25.



26.



27.



28. **ANGLE MEASURE** Make a prediction about how you could use trigonometric ratios to find angle measures in a triangle.

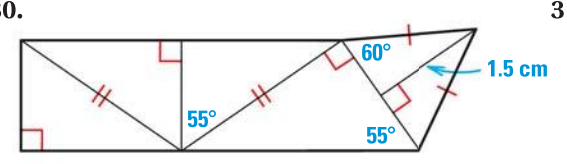
29. ★ **MULTIPLE CHOICE** In $\triangle JKL$, $m\angle L = 90°$. Which statement about $\triangle JKL$ *cannot* be true?

Ⓐ $\sin J = 0.5$ Ⓑ $\sin J = 0.1071$

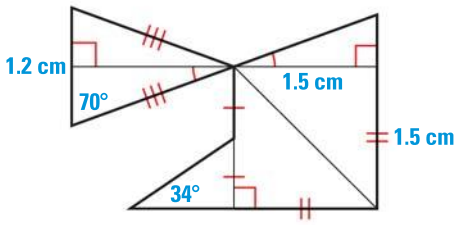
Ⓒ $\sin J = 0.8660$ Ⓓ $\sin J = 1.1$

PERIMETER **Find the approximate perimeter of the figure.**

30.



31.



32. **CHALLENGE** Let A be any acute angle of a right triangle. Show that (a) $\tan A = \frac{\sin A}{\cos A}$ and (b) $(\sin A)^2 + (\cos A)^2 = 1$.

PROBLEM SOLVING

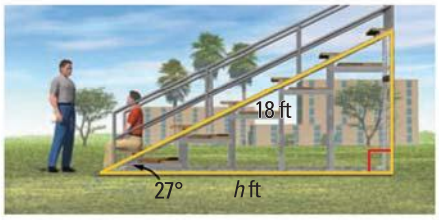
EXAMPLES 4 and 5
on pp. 475–476
for Exs. 33–36

33. AIRPLANE RAMP The airplane door is 19 feet off the ground and the ramp has a 31° angle of elevation. What is the length y of the ramp?

@HomeTutor for problem solving help at classzone.com

34. BLEACHERS Find the horizontal distance h the bleachers cover. Round to the nearest foot.

@HomeTutor for problem solving help at classzone.com



35. ★ SHORT RESPONSE You are flying a kite with 20 feet of string extended. The angle of elevation from the spool of string to the kite is 41°.

a. Draw and label a diagram to represent the situation.

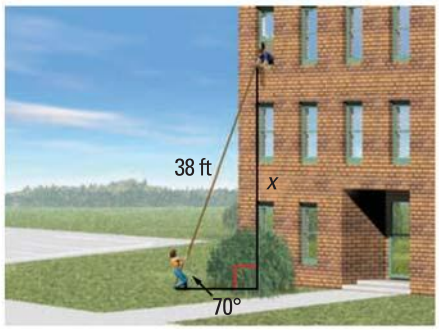
b. How far off the ground is the kite if you hold the spool 5 feet off the ground? *Describe* how the height where you hold the spool affects the height of the kite.

36. MULTI-STEP PROBLEM You want to hang a banner that is 29 feet tall from the third floor of your school. You need to know how tall the wall is, but there is a large bush in your way.

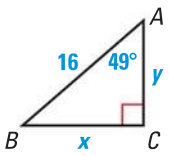
a. You throw a 38 foot rope out of the window to your friend. She extends it to the end and measures the angle of elevation to be 70°. How high is the window?

b. The bush is 6 feet tall. Will your banner fit above the bush?

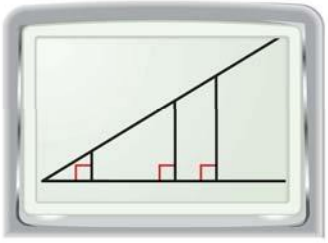
c. What If? Suppose you need to find how far from the school your friend needs to stand. Which trigonometric ratio should you use?



37. ★ SHORT RESPONSE Nick uses the equation $\sin 49° = \frac{x}{16}$ to find BC in $\triangle ABC$. Tim uses the equation $\cos 41° = \frac{x}{16}$. Which equation produces the correct answer? *Explain*.



38. TECHNOLOGY Use geometry drawing software to construct an angle. Mark three points on one side of the angle and construct segments perpendicular to that side at the points. Measure the legs of each triangle and calculate the sine of the angle. Is the sine the same for each triangle?



39. **MULTIPLE REPRESENTATIONS** You are standing on a cliff 30 feet above an ocean. You see a sailboat on the ocean.

a. Drawing a Diagram Draw and label a diagram of the situation.

b. Making a Table Make a table showing the angle of depression and the length of your line of sight. Use the angles 40°, 50°, 60°, 70°, and 80°.

c. Drawing a Graph Graph the values you found in part (b), with the angle measures on the x-axis.

d. Making a Prediction Predict the length of the line of sight when the angle of depression is 30°.

40. **ALGEBRA** If $\triangle EQU$ is equilateral and $\triangle RGT$ is a right triangle with $RG = 2$, $RT = 1$, and $m\angle T = 90°$, show that $\sin E = \cos G$.

41. **CHALLENGE** Make a conjecture about the relationship between sine and cosine values.

a. Make a table that gives the sine and cosine values for the acute angles of a 45°-45°-90° triangle, a 30°-60°-90° triangle, a 34°-56°-90° triangle, and a 17°-73°-90° triangle.

b. Compare the sine and cosine values. What pattern(s) do you notice?

c. Make a conjecture about the sine and cosine values in part (b).

d. Is the conjecture in part (c) true for right triangles that are not special right triangles? *Explain*.

MIXED REVIEW

Rewrite the equation so that x is a function of y. *(p. 877)*

42. $y = \sqrt{x}$ **43.** $y = 3x - 10$ **44.** $y = \frac{x}{9}$

PREVIEW
Prepare for Lesson 7.7 in Exs. 45–47.

Copy and complete the table. *(p. 884)*

45.

x	$\sqrt{x}$
?	0
?	1
?	$\sqrt{2}$
?	2
?	4

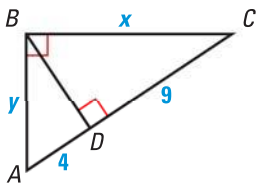
46.

x	$\frac{1}{x}$
?	1
?	$\frac{1}{2}$
?	3
?	$\frac{2}{7}$
?	7

47.

x	$\frac{2}{7}x + 4$
?	0
?	2
?	6
?	8
?	10

48. Find the values of x and y in the triangle at the right. *(p. 449)*



PROBLEM SOLVING WORKSHOP
LESSON 7.6

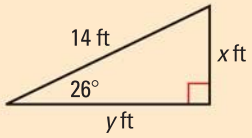
Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 476

MULTIPLE REPRESENTATIONS You can use the Pythagorean Theorem, tangent ratio, sine ratio, or cosine ratio to find the length of an unknown side of a right triangle. The decision of which method to use depends upon what information you have. In some cases, you can use more than one method to find the unknown length.

PROBLEM

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and base of the ramp.



METHOD 1

Using a Cosine Ratio and the Pythagorean Theorem

STEP 1 **Find** the measure of the third angle.

$26° + 90° + m\angle 3 = 180°$ Triangle Sum Theorem

$116° + m\angle 3 = 180°$ Combine like terms.

$m\angle 3 = 64°$ Subtact 116° from each side.

STEP 2 **Use** the cosine ratio to find the height of the ramp.

$\cos 64° = \frac{\text{adj.}}{\text{hyp.}}$ Write ratio for cosine of 64°.

$\cos 64° = \frac{x}{14}$ Substitute.

$14 \cdot \cos 64° = x$ Multiply each side by 14.

$6.1 \approx x$ Use a calculator to simplify.

▶ The height is about 6.1 feet.

STEP 3 **Use** the Pythagorean Theorem to find the length of the base of the ramp.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ Pythagorean Theorem

$14^2 = 6.1^2 + y^2$ Substitute.

$196 = 37.21 + y^2$ Multiply.

$158.79 = y^2$ Subtract 37.21 from each side.

$12.6 \approx y$ Find the positive square root.

▶ The length of the base is about 12.6 feet.